RECENT ADVANCES IN STATISTICAL AND SCALING ANALYSES OF FIELD SCALE PERMEABILITIES

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ABSTRACT

Many earth and environmental variables appear to be self-affine (monofractal) or multifractal with Gaussian or heavy-tailed distributions. The literature considers self-affine and multifractal types of scaling to be fundamentally different, the first arising from additive and the second from multiplicative random fields or processes. Our recent work demonstrates theoretically and numerically that square or absolute increments of samples from truncated fractional Brownian motion (tfBm) exhibit apparent multifractality at intermediate ranges of separation scales, with breakdown in power-law scaling at small and large lags as is commonly exhibited by data. The same is true of samples from sub-Gaussian processes subordinated to tfBm with heavy tailed subordinators such as log-normal or Lévy, the latter leading to spurious behavior. It has been established empirically that, in numerous cases, the range of lags exhibiting power-law scaling can be enlarged significantly, at both ends of the spectrum, via a procedure known as Extended Self-Similarity (ESS). Whereas the literature tends to associate extended and nonlinear power-law scaling with multifractals or fractional Laplace motions, we show that (a) ESS of data having a normal frequency distribution is theoretically consistent with (Gaussian) truncated (additive, self-affine, monofractal) fractional Brownian motion (tfBm), the latter being unique in predicting a breakdown in power-law scaling at small and large lags, and (b) nonlinear power-law scaling of data having either normal or heavy-tailed frequency distributions is consistent with samples from sub-Gaussian random fields or processes subordinated to tfBm, stemming from lack of ergodicity which causes sample moments to scale differently than do their ensemble counterparts. We illustrate these findings through the analysis of the scaling behavior of a log permeability data set showing heavy-tailed frequency distributions in three spatial dimensions. The set consists of 1-m scale pneumatic packer test data from six vertical and inclined boreholes spanning a decameters scale block of unsaturated fractured tuffs near Superior, Arizona. We (i) demonstrate that the above data set is consistent with sub-Gaussian random fields subordinated to tfBm and (ii) provide maximum likelihood estimates of parameters characterizing the corresponding Lévy stable subordinators and tfBm functions.

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